

Thoughts on Bargaining over Attribute Pairs

1. We assume a bargaining between two players with number of attributes, $X_1 \dots X_k$, $k > 2$. The measurement of attributes is in their original metrics or by bargainers' value functions, $f_i(X_k)$. The problem is easier to describe using the original attribute metrics such that all pairs of attributes (X_j, X_k) exhibit strict (linear) tradeoffs. For the following example we use three attributes: for NASCAR the three attributes are fuel economy, acceleration (proxy, horsepower or torque), and curb weight.¹ The racing team needs to configure their car, and two sides may emerge, say owners/drivers vs engineers/technicians.

2. To initially illustrate, let X_1 be fuel economy, $X_2 \Rightarrow$ acceleration, $X_3 \Rightarrow$ curb weight. Let P_1 be owners/drivers and P_2 engineers/technicians. Each side establishes an ordinal and transitive order of attribute preferences, i.e., $p_i \rightarrow (X_j > X_{j+1} > X_{j+2})$. Therefore, each side (P_1 or P_2) can use or configure six possible orders of attribute preferences. For instance, P_1 and P_2 could respectively establish the preference orders $p_1 \rightarrow (X_3 > X_2 > X_1)$, and $p_2 \rightarrow (X_1 > X_2 > X_3)$. Because attributes exhibit strict tradeoffs, we assume bargainers negotiate over attribute pairs: Hence, the preferences of P_1 and P_2 give the possible (non-duplicative) negotiations over the three pairs of attributes.

A. Symmetric Conflict. For the p_1 and p_2 example above, we obtain the following preferences for attribute tradeoffs or comparisons (Read down for pair comparisons over the three negotiations):

y: $X_1(P_2)$ $X_1(P_2)$ $X_2(P_2)$

x: $X_2(P_1)$ $X_3(P_1)$ $X_3(P_1)$

I call this particular negotiation *Symmetric Conflict* because each player prefers an attribute opposite to his/her opponent's preference (see Rose, Menasco, and Curry). There are two (2) orderings of these particular attribute negotiations with X_2 as the second most preferred attribute by both players. If $p_1 \rightarrow (X_1 > X_2 > X_3)$ and $p_2 \rightarrow (X_3 > X_2 > X_1)$, which is the reverse of original preferences, then player preferences for attribute comparisons are simply the opposite of the (x, y) example above.

Another order is $p_1 \rightarrow (X_2 > X_3 > X_1)$ and $p_2 \rightarrow (X_1 > X_3 > X_2)$. Preferences yield the following (again, read down):

y: $X_1(P_2)$ $X_1(P_2)$ $X_2(\mathbf{P}_1)$

x: $X_2(P_1)$ $X_3(P_1)$ $X_3(\mathbf{P}_2)$

This is still *symmetric conflict*, because the assignment of the (x, y) coordinates to the pair (X_2, X_3) is arbitrary (note the **bold type** above indicating preferences for P_1 and P_2). There are four (4) orderings of these negotiations based on X_1 or X_3 as the second most preferred attribute by both players.

B. Asymmetric Conflict. Another order is $p_1 \rightarrow (X_1 > X_3 > X_2)$ and $p_2 \rightarrow (X_1 > X_2 > X_3)$, yielding the preferences:

y: $X_1(P_1, P_2)$ $X_1(P_1, P_2)$ $X_2(P_2)$

x: X_2 X_3 $X_3(P_1)$

In other words, both players prefer X_1 over X_2 and X_3 . There are twenty-four (24) asymmetric kinds of combinations (for which bargainers prefer one or two of the same

attributes). For example, let us take the orderings $p_1 \rightarrow (X_2 > X_3 > X_1)$ and $p_2 \rightarrow (X_1 > X_2 > X_3)$. These orderings yield preferences:

y: $X_1(P_2) \ X_1(P_2) \ X_2(P_1, P_2)$

x: $X_2(P_1) \ X_3(P_1) \ X_3$

Both players prefer X_2 over X_3 , but they have opposing preferences for X_1 vs X_2 and X_1 vs X_3 . I call these 24 combinations *asymmetric conflict* because both bargainers prefer the same attribute for one or two of the three combinations (but not all three).

C. Agreement. Bargaining conflict is absent when players agree on order of preferences, p_i ; for instance, $p_1 = p_2 \rightarrow (X_1 > X_2 > X_3)$. There are six (6) of these non-duplicative orderings.

3. Solutions. We assume bargainers process in a lexicographic way to determine the first (X_j, X_k) comparison. In other words, they each choose the most important attribute to begin negotiations. At the extreme each player would like to maximize X_j or X_k . The number of non-duplicative comparisons (y, x) is only peripherally relevant (and there are 36 if I counted correctly), in the sense it is the *type* conflict determines possible outcomes.

A. Symmetric Conflict. In the case of symmetric conflict, a natural solution would be to compromise which is the midpoint on the frontier formed by the tradeoffs between the two attributes. If this solution exists for bargainers' most preferred attributes defined by the pair (X_j, X_k) , then a midpoint solution also exists for the other two comparisons (see attached graph). In fact, this is the only solution that yields a pair of points on *each* of the three attribute frontiers.² If other solutions emerge, those solutions tend to favor one party based on the original ordering of bargainers' preferences.³

B. Asymmetric Conflict. Because players prefer the same attribute in at least one (X_j, X_k) comparison, solutions are dependent on the preference ordering of attributes by players. If players prefer the same attribute in a lexicographic ordering of attribute comparisons, then players should select max X_i and proceed to second stage comparisons as outlined in footnote 3. Alternatively, if players prefer opposite attributes in the first lexicographic order, then a midpoint solution should occur for all three attribute comparisons as in symmetric conflict.

C. Agreement. In this case, players prefer the same ordering of attributes as identified in 2C, above. One of the three pairs of comparisons is a priori irrelevant, because players prefer the same attributes over the three comparisons. They maximize the most important (common) attribute and minimize the remaining two attributes. Consider the ordering given in 2C: $p_1 = p_2 \rightarrow X_1 > X_2 > X_3$ yields preferences,

y: $X_1(P_1, P_2) \ X_1(P_1, P_2) \ X_2(P_1, P_2)$

x: $X_2 \ \ X_3 \ \ X_3$

Hence, X_1 is a maximum while X_2 and X_3 are minimums. The comparison of X_2 vs. X_3 is nonessential.

4. Research Hypotheses. Statements of research hypotheses derive from the solution concepts discussed in 3. above.

A. Symmetric Conflict. Since compromise appears to be a reasonable arithmetic solution, then midpoint outcomes should be more prevalent in this kind of bargaining.

B. Asymmetric Conflict. This type of conflict will produce either (1) compromise (both p_1 and p_2 prefer different attributes) or (2) players prefer the same attribute and therefore maximize a particular X_i , reducing the other two attributes to minimums.

C. Agreement. Players maximize their (common) most important attribute and minimize the other two attributes.

Notes.

1. For this example, acceleration of the car is important at the apex of a curve where the driver needs to move the car to a straight away. Obviously, the greater the acceleration, the less the fuel economy, and the more the car weighs greater acceleration is required; just as weight is inversely related to fuel economy: The goal of every team is to minimize pit-stops. However, acceleration is dependent on engine torque which may be independent of curb weight. Hence, there is some confounding here, and I'm not convinced that in 'real life experiments' that there are actually natural trade-offs for any more than two attributes. I don't know how much there is in the literature on this subject.

On the other hand, a researcher could explicate a value or utility function for attributes for each side (per Keeney and Raiffa) and sides could sequentially negotiate over each of the k attributes in utility space. This has the advantage of accommodating a variety of frontiers (linear and non-linear) much like what is in the prevalent literature where bargainers combine utilities to reach a solution. The model I am proposing here is simple (parsimonious) but perhaps not realistic. This mimics models of individuals processing by attributes vs processing by brands.

2. A simultaneous solution for all three frontiers does not exist except at midpoints, even if one pair of attributes exhibits a positive, linear relationship (representing the confounding problem noted in footnote 1). This is proved easily algebraically. Footnote 3 presents examples using $\max X_i$ and EE or EI like solutions that require an initial negotiation and the choice of a second stage comparison. General outcomes should also hold for nonlinear attribute frontiers. I investigated the quarter circle frontier, and *one* (X_j, X_k) comparison is also irrelevant for all solutions other than the midpoint for the first pair (read footnote 3).

3. Consider the first example from A. above: The first negotiation should be the pair $[(X_1) P_2]$ vs $[(X_3) P_1]$ since these are the most preferred attributes by players. Assume P_1 capitulates so they settle on $\max X_1$ and $\min X_3$. The other two comparisons might have equal probability of selection for the next negotiation, because X_2 is each player's second preferred attribute in combination with their most preferred (X_1 or X_3). Given $\max X_1$, then we have $\min X_2$ for that negotiation (comparison). This renders the comparison X_2, X_3 frivolous or nonessential, because both X_2 and X_3 reduce to a minimum and that point does not exist on the frontier. P_2 wins by virtue of getting $\max X_1$, while P_1 only obtains $\min X_2$ and $\min X_3$. Alternatively, if players next go to the X_2, X_3

comparison, that gives max X_2 for P_2 by virtue of min X_3 . The X_1, X_2 comparison is nonessential: both max X_1 and max X_2 cannot exist together. Again, P_2 wins, but P_1 obtains some solace by earning max X_2 (his/her second most preferred attribute).

Consider the second example from A. above: Assume they settle on max X_1 (favoring P_2). The next comparison involves X_3 ; and against (X_1, X_3) , X_3 must be a minimum: P_1 loses. Since X_2 (favored by P_1) is a minimum, X_3 (favored by P_2) must be a maximum in the (X_2, X_3) comparison and again P_1 loses.

Consider a solution that conforms to EE or EL in the attribute spaces from the first example in A. above: From the graph we see that an equal loss solution occurs at (58, 18) in the X_1, X_3 coordinates. This translates to a solution of (58, 25) for the comparison of X_1, X_2 , and X_2, X_3 becomes irrelevant. Using X_2, X_3 as the next stage comparison, the value of X_2 is 45 (given $X_3 = 18$). Again, the players might prefer this comparison for the second stage, because it yields the highest value for X_2 as both players' second preferred attribute.

Obviously, the zero origin I used in the graph is arbitrary, because min X_i is just a scale transformation. So, minimum acceleration in the NASCAR example would not go to zero but rather a nonzero value given maximum curb weight.

Appendix: Key for the Excel Chart.

The three points on each graph (exclusive of endpoints) represent solutions for each of the three X_j, X_k comparisons. Solutions are:

- (1) EL defined as equal losses of units (utility) from each player's maximum score.
- (2) Midpoint defined as the exact midpoint between any X_j, X_k tradeoff.
- (3) EE defined as equal earnings of units (utility) from each player's minimum score.

Blue graph. X_1 vs X_2 : EL = (51, 31); Midpoint = (45, 35); EE = (39, 39)

Red graph. X_1 vs X_3 : EL = (58, 18); Midpoint = (45, 25); EE = (32, 32)

Green graph. X_2 vs X_3 : EL = (41, 21); Midpoint = (35, 25); EE = (29, 29)